TDT4120 Algorithms and Data Structures

Examination, December 19, 2022, 15:00–19:00

Academic contact Magnus Lie Hetland
Support material code E

Problems

1 What is the running time of DIJKSTRA, with a binary heap as priority queue?

Give your answer in O notation. You may assume $|E| = \Omega(V)$.

2 $Q = \langle 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 \rangle$ is an array used to implement a FIFO queue.

Perform the following procedure.

```
Q.head = 1
Q.tail = 2
ENQUEUE(Q, 1)
ENQUEUE(Q, 2)
Q.head = 9
Q.tail = 10
ENQUEUE(Q, 3)
ENQUEUE(Q, 4)
```

What does Q look like afterward?

3 Why is memoization not useful when using the design method *divide and conquer*?

4 What is *chaining* used for?

You need not explain how it works.

5 Give lower and upper asymptotic bounds for the expression $n + \Theta(n^2) + O(n^3)$.

6 Simplify the expression $\Omega(n + \Theta(n^2) + O(n^3))$.

7 Solve the recurrence $T(n) = 4T(n/2) + n^2 \lg n$. Give your answer in Θ notation.

8 Starting with an empty binary search tree, insert the following values, in order, using TREE-INSERT:

$\langle 7, 1, 0, 5, 4, 8, 3, 2, 9, 6 \rangle$

Then perform INORDER-TREE-WALK on the root node of the resulting tree. What does the algorithm print out?

Your answer should only be the output of the algorithm.

9 The following matrix is the weight matrix of a weighted, directed graph:

$$W \quad \begin{array}{c|c|c|c|c|} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 8 & \infty & 2 \\ \hline 2 & 1 & 0 & 5 & 1 \\ \hline 3 & 7 & \infty & 0 & 1 \\ \hline 4 & 5 & 8 & 6 & 0 \\ \hline \end{array}$$

Perform SLOW-APSP on the graph. What is $l_{3,1}^{(2)}$?

10 Assume that you add a check to BELLMAN-FORD that terminates the algorithm if no distance estimates change in the course of one iteration. What is the resulting total running time, in the best case, if you assume there are paths from the starting vertex to all others? Explain briefly.

11 What is the minimum and maximum number of elements in a binary heap with height h?

12 What does the integrality theorem say? Explain briefly in your own words.

13 What is residual capacity and how does one calculate it? Explain briefly.

14 Your friend Smartnes thinks *graph isomorphism* is at least as hard as *factoring*. To establish this, she intends to show that solutions to one of the problems can, with some extra computation, be used to solve the other one. Which problem's solution must then be used to solve the other one, and why does this lead to the desired conclusion? Explain.

Algorithm 1 Lurvik's version of *randomized select*

```
RANDOMIZED-SELECT(A, p, r, i)
if r ⩽ p
    return A[p]
q = RANDOMIZED-PARTITION(A, p, r)
k = q − p + 1
if i == k
    return A[q]
elseif i < k
    RANDOMIZED-SELECT(A, p, q − 1, i)
    RANDOMIZED-SELECT(A, q + 1, r, i − k)
```

15 Your friend Lurvik has tried to write down pseudocode for *randomized select*, from memory. The result (Algorithm 1) is not quite right. Describe what must be fixed for the algorithm to be correct.

16 Which problem does Algorithm 1 solve, if it is called as follows, where $A[1:n]$ is an array of numbers?

```
RANDOMIZED-SELECT(A, 1, n, 0)
```

Explain briefly.

17 Your friend Gløgsund has created two versions of the Ford–Fulkerson method, where she uses DIJKSTRA and TRANSITIVE-CLOSURE, respectively, to find augmenting paths. Which of these two methods is guaranteed to find maximum flow in polynomial time? Explain briefly.

Assume that $w(u, v) = 1$ for all edges (u, v) in the residual network, and that Gløgsund maintains a Π matrix with predecessors in TRANSITIVE-CLOSURE to find the actual paths.

18 We say a woman and a man are *meant for each other* if they end up together in every possible stable matching. Design an efficient algorithm that decides if a woman and a man are meant for each other.

19 How can we solve the subset-sum problem in polynomial time if the target is given in the unary numeral system?

In the unary numeral system, k is represented as a string $111\cdots1$ of length k.

20 A kingdom consists of several regions. The king wants to build a wall that encompasses one or more of the regions, including the one containing the royal castle. The building costs vary with the terrain, and the king has asked you to find the cheapest solution. How would you proceed?

You may assume that the wall follows region borders.